MA’RUZA

VEKTORLAR VA ULAR USTIDA CHIZIQLI AMALLAR.VEKTORNING KOORDINATALARI. BAZIS. IKKI VERTORNING SKALYAR KO’PAYTMASI.

Ma’ruza rejasi

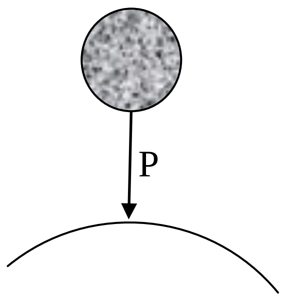
1. Skalyar va vektor miqdorlar.
2. Vektorlar ustida chiziqli amallar (qo’shish, ayirish va skalyarga ko’paytirish).
3. Vektorning o’qdagi proyeksiyasi.
4. Vektorning yo’naltiruvchi burchaklari (yo’naltiruvchi kosinuslari).
5. Tekislikdagi va fazodagi vektorlar bazisi. Ortogonal va ortonormal bazislar. Vektorlarning tashkil etuvchilari.

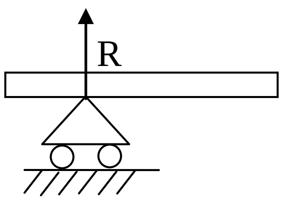
Tayanch so’z va iboralar: skalyar miqdor, vektor miqdor, proyeksiya (soya), koordinatalar sistemasi, bazis, birlik vektor, ortogonal vektorlar, modul, yo’naltiruvchi burchak, vektorning koordinatalari, vektorning komponentlari (tashkil etuvchilari).

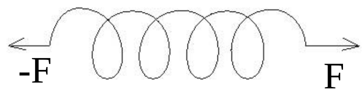
Ta’sir muhitlarni o’rganar ekanmiz, ko’pincha ikki xil miqdorni bir-biridan farq qilamiz. Ulardan biri **skalyar** miqdor. Ikkinchi esa **vektor** miqdordir.

Vektor – bu ham miqdor o’lchoviga, hamda yo’nalishga ega bo’lgan kattalik.

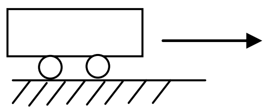
Misol: Kuch (og’irlik kuchi, reaksiya kuchi, elastiklik kuchi),







yoki tezlik va tezlanish



v-harakat yo’nalishi tomonga yo’nalgan.

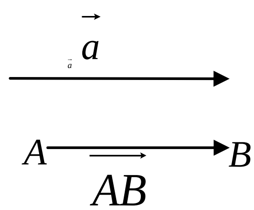
Ana shu misol qilganimiz kuchlar fizik qonunlarda :

a) $P=mg$ (g – erkin tushish tezlanishi. m - jism massasi, P-shu jismning yerga tortish kuchi);
b) $R=-kF$ F tayanchga bosuvchi aktiv kuch (masalan yuk), R- reaksiya kuchi;
c) F va $-F$ lar prujinani ikki yoqqa cho’zuvchi kuchlar ($F=\rho slg$). g- qattiqlik koeffisiyenti. ρ – zichlik. l – prujina uzunligi, s – ko’ndalang kesim yuzasi;
d) $J=\dfrac{mv^2-mv_o^2}{2}$ - inertsiya kuchi

formulalar bilan ifodalanadiki, bunda ishtrok etayotgan m – massa, ρ – zichlik, s – ko’ndalang kesim yuzi, … kabilar hech qanday yo’nalishga ega emas, ammo ma’lum bir kattalikni ifodalovchi faktor.

Masalan $P=mg$ – ni olsak P- kuch miqdor va u yer markaziga yo’nalgan. Ammo jism harakatdami yoki tinch turibdimi, baribir undagi modda miqdori o’zgarmaydi. Ya’ni jismdagi modda miqdori – skalyar miqdordir.

Vektor $\vec{a}, \vec{b}, \vec{c}, \dots$ yoki $\overrightarrow{AB}, \overrightarrow{CD}, \dots$ kabi harflar bilan belgilanib, kabi strelka bilan ifodalanadi.

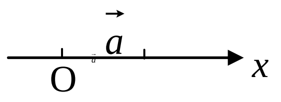


A nuqta vektorning boshi, B- esa oxiri (uchi) strelka esa uning yo’nalishi deyiladi.

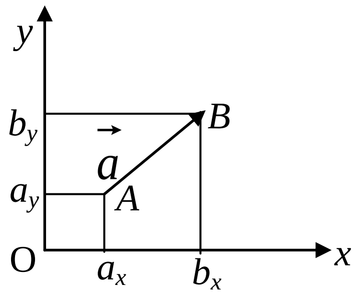
U yotgan to’g’ri chiziq vektorning yo’naltiruvchi chizig’i (o’qi) deb yuritiladi.

Koordinatalar sistemasida: vektor o’zining koordinatalari bilan beriladi:

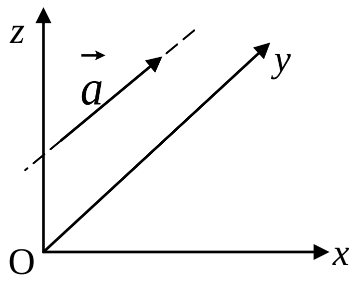
a)



b) $\overrightarrow{AB} = \vec{a} = \{a, b\}$ yoki $\left|\overrightarrow{AB}; A(a_x, a_y), B(b_x, b_y)\right|$



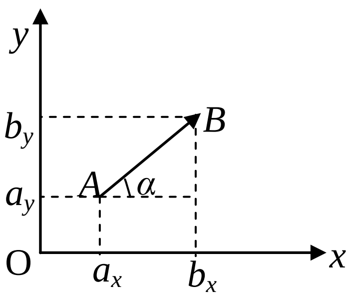
c)



Bunday berilishlarni vektorning dekart koordinatalari sistemasidagi berilishi deymiz.

Xususan $\overrightarrow{AB} = \{b_x - a_x, b_y - a_y\}$ ni $\overrightarrow{AB}$ vektor uchlari (A va B)ning koordinatlari bilan berilgan deymiz. $\vec{a} = \{a_x,\ a_y\}$ni esa $\vec{a}$ -vektor o’zining koordinata o’qlaridagi soyalari (proeksiyalari) bilan berilgan deymiz.

Albatta biz o’rta maktabda nuqtaning geometrik o’rnini aniqlashni o’rganganmiz.



Keling tushunishni osonlashtirish uchun ikki o’lchovli (*x*O*y*) to’gri burchakli koordinatalar sistemasini olib, bu kattaliklarni yasaymiz va miqdorlar munosabatini aniqlaymiz.

Berilgan $A(a_x a_y)$ *va* $B(b_x b_y)$; 1) $\overrightarrow{AB}$ yasalsin. 2) Uning proyeksiyalari va moduli topilsin.

Eslatma. *A* nuqtani yasash uchun O*x*- o’qidan a_x kesma ajratib,uning oxiridan Oy-ga parallel shtrix O*y* o’qidan a_y kesmani ajratib, uni oxiridan O*x*-ga parallel chiziqlar chiqaramiz, ularning kesushuvi yagona bo’lib, *A*nuqtaning o’rnini belgilaydi. Shunday qilib, *B* ni yasaymiz. Ularni to’gri chiziq kesmasi bilan tutashtirib, *B* tomonga strelka qo’yamiz, $\overrightarrow{AB}$ vektor yasaldi. Ordinatalarga parallel shtrix chiziqlar bilan ΔABD ni yasaymiz. Mulohasa qilib ko’raylik.

1. Δ *ABD*- to’g’ri burchakli.

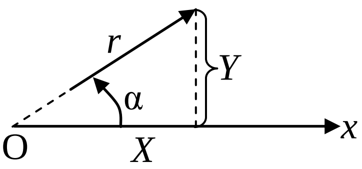
2. $\angle$ *BAD* ni - α deb belgilasak va $\overrightarrow{AB}$ ning O*x* o’qiga og’ish burchagi.

3. $\left|\overrightarrow{AB}\right| = r$ bilan $\overrightarrow{AB}$ vektor uzunligini belgilasak va uni vektorning moduli deb atasak: α - og’ish burchagini O*x* ga nisbatan $\overrightarrow{AB}$ ni yo’naltiruvchi burchagi deb atasak, quyidagilarga ega bo’lamiz:

1. $\begin{array}{l} X = b_x - a_x = np_{Ox}\vec{AB} = rCosa \\ Y = b_y - a_y = np_{Oy}\vec{AB} = rSina \end{array} \Bigg| \Rightarrow r = \left|\vec{AB}\right| = \sqrt{(b_x - a_x)^2 + (b_y - a_y)^2}$

ko'rinib turibdiki, α va r – berilgan $\overrightarrow{AB}$ vektorni yagona ravishda ifodalaydi.

2. Demak, vektor $r = (r, a)$ -ko'rinishda ham berilar ekan .



Bunday berilishga qutb koordinatalar sistemasida vektorning berilishi deb ataymiz. U vaqtda: $r = \sqrt{x^2 + y^2}$

3. $\begin{array}{l} Cosa = \dfrac{X}{r} = \dfrac{X}{\sqrt{x^2 + y^2}} \\ Sina = \dfrac{Y}{r} = \dfrac{Y}{\sqrt{x^2 + y^2}} \end{array} \quad \left. \begin{array}{l} x = rCosa \\ y = rSina \end{array} \right\}$

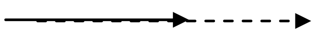
Formulalarni koordinatalar almashtirish formulalari deb ataymiz.

4. Eslatma : $AB = \sqrt{(b_x - a_x)^2 + (b_y - a_y)^2}$ formula ikki nuqta orasidagi masofa (kesmaning uzunligi)ni topish formulasi deb ham yuritishadi. Vektorlar maydon tushunchasini vujudga keltirish uchun quyidagi tushunchalarni kiritamiz.

a) Moduli (uzunligi) 1ga teng vektorni birlik vektor deb ataymiz va ularni $\vec{i}, \vec{j}, \vec{k}$ deb ataymiz. (|i|=|j|=|k|=1).

b) Agar $\vec{i} \perp \vec{j}, \vec{j} \perp \vec{k}, \vec{k} \perp \vec{i}$ bo'lsa bu vektorlarni ortogonal vektorlar deb ataymiz

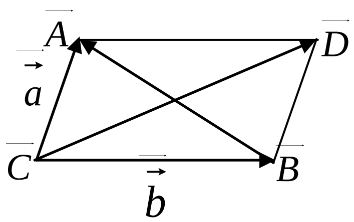
в) Vektorni k skalyar (songa) ko'paytirganda, avvalgi $(\vec{a})$ vektorga nisbatan skalyar k barobar uzaygan, ammo yo'nalishi avvalgicha qolgan vektor hosil bo'ladi.



$\vec{a} \| \vec{a}_1$

$k\vec{a} = \vec{a}_1$

g) **Eslatma.** O'rta maktabdan bilamizki, ikki vektor qo'shilganda ulardan parallelogramm yasalib, ular qo'yilgan nuqtadan chiqqan diogonalni esa ularning yig'indisi deb, ikkinchi diagonalni esa ularning ayirmasi deb olinadi.



$\vec{a} + \vec{b} = \vec{c} = \overrightarrow{CD}$

$\vec{a} - \vec{b} = \vec{d} = \overrightarrow{BA}$

E'tibor qiling: $\begin{array}{l} \vec{b} \| \overrightarrow{AD}, \ |\vec{b}| = |\overrightarrow{AD}| \\ \vec{a} \| \overrightarrow{BD}, \ |\vec{a}| = |\overrightarrow{BD}| \end{array}$

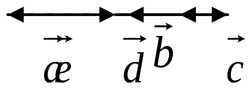
Demak, $\overrightarrow{AD}$ vektor berilgan $\vec{b}$ vektorga kollinear (bir tekislikda yotuvchi a yo'naltiruvchi chiziqlari o'zaro parallel) vektorlardir.

Chizmadan ko'rinib turibdi: $\vec{a}+\vec{b}=\vec{a}+\overrightarrow{AD}=\vec{c}$ ekan.

1. Demak, ikki vektorni qo'shsish uchun ularni birin -ketin qo'yib, ochiq uchlarini tutashtirish lozim ekan.

2. Ikki vektorni biridan ikinchisini ayirish uchun ularni $\vec{a}-\vec{b}=\vec{a}+\left(-\vec{b}\right)$ shaklda birin-ketin qo'yib, ochiq uchlarini tutashtirish lozim ekan.

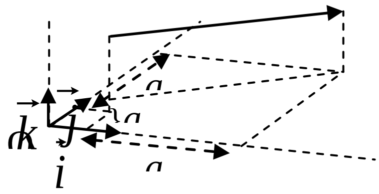
Demak, *n*ta vektor berilganda ham, bu amal qo'shish yoki ayirishda muammo vujudga keltirmas ekan.



$$\vec{e}=\vec{a}+\vec{b}+\vec{d}+\vec{c}$$

Koordinatalari bilan berilgan vektorlarni qo'shish va ayirish.

$\vec{a}=\{a_x,a_y,a_z\}$ $\vec{b}=\{b_x,b_y,b_z\}$



$$\vec{c}=\vec{a}+\vec{b}=\{a_x+b_x,\ a_y+b_y,\ a_z+b_z\} \qquad \vec{d}=\vec{a}-\vec{b}=\{a_x-b_x,a_y-b_y,a_z-b_z\}$$

Ta'rif : ($\vec{i},\ \vec{j},\ \vec{k}$) -ortonormal vektorlar sistemasi bo'lsa, uni bazis deb ataymiz va uni **vektor koordinatalar sistemasi** deb qabul qilamiz.

$$\vec{a}=\{a_x,a_y,a_z\}$$

$$\vec{a}_x=a_x\vec{i},\quad \vec{a}_y=a_y\vec{j},\quad \vec{a}_z=a_z\vec{k},\quad \vec{a}=a_x\vec{i}+a_y\vec{j}+a_z\vec{k}$$

ifodalar $\vec{a}$ vektorning ($\vec{i},\ \vec{j},\ \vec{k}$)-ortonormal bazisdagi berilishi deyiladi. Chizmaga yaxshiroq e'tibor berilsa, $\vec{a}$ qolgan uchta $a_x\vec{i},\ a_y\vec{j},\ a_z\vec{k}$ vektorlar yig'indisi (qo'shiluvidan) hosil bo'lgan vektordir.

Demak, a_x,a_y,a_z-larni $\vec{a}$ vektorning $\vec{i},\ \vec{j},\ \vec{k}$ yo'nalishlardagi proyeksiyasi deyish bilan birga a_xi,a_yj,a_zk vektorlarni uning tashkil etuvchilari deb ayta olamiz.

Misol : $\vec{a}=a_x\vec{i}+a_y\vec{j}+a_z\vec{k}$ | berilgan $\vec{a}+\vec{b}-$ni tashkil etuvchilar i bilan yozing.

$\vec{b}=b_x\vec{i}+b_y\vec{j}+b_z\vec{k}$ |

Javob: $\vec{a}+\vec{b}=\{a_x+b_x,a_y+b_y,a_z+b_z\}=(\text{yoki})=(a_x+b_x)\vec{i}+(a_y+b_y)\vec{j}+(a_z+b_z)\vec{k}$ deb yoza olamiz.

Albatta, yo'naltiruvchi burchaklar ortonormal bazis ham o'z ma'no-e'tiborini saqlaydi.

$$\left(\vec{i}\wedge\vec{a}\right)=\alpha,\left(\vec{j}\wedge\vec{a}\right)=\beta,\left(\vec{k}\wedge\vec{a}\right)=\gamma,\ a_x=np_i\vec{a},\ a_y=np_j\vec{a},\ a_z=np_k\vec{a}$$

Teorema : $Cos^2\alpha+Cos^2\beta+Cos^2\gamma=1$

Isbot : $Cos\alpha=\dfrac{a_x}{r},\cos\beta=\dfrac{a_y}{r},\cos\gamma=\dfrac{a_z}{r}$

$$Cos^2\alpha+\cos^2\beta+\cos^2\gamma=\frac{a_x^2}{r^2}+\frac{a_y^2}{r^2}+\frac{a_z^2}{r^2}=\frac{a_x^2+a_y^2+a_z^2}{r^2}=\frac{r^2}{r^2}=1$$

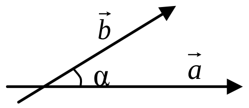
Vektorni skalyarga ko'paytirsak, skalyar barobar o'zgargan (miqdoran) va yonalishi avvalgicha qolgan yangi vektor vujudga keladi.

Vektorga mexanik ma'no bersak,biz tezlik, tezlanish va kuchlarni eslaymiz V ham tezlik, $V_1=kV$ ham tezlik.

Vni k (skalyar) songa ko'paytirish bilan yana yangi tezlikni oldik, bunda $V_1 \| kV : |V_1| = k|V|$.

Bu darsimizda ikkita vektorni (tezlikni) o'zaro ko'paytirishga urinib ko'ramiz.

Albatta biz ko'paytirish natijasiga qarab ularni nomlaymiz. Aytaylik $\vec{a}$ va $\vec{b}$ -vektorlar berilgan bo'lsin.



(i, j) –ortonormal bazis bo'lsin.

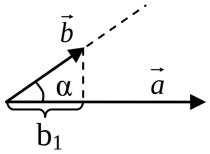
$\vec{a}=a_1\vec{i}+a_2\vec{j}$

$\vec{b}=b_1\vec{i}+b_2\vec{j}$

1-ta'rif . Ikki vektor orasidagi burchak deb, ularning yo'naltiruvchi chiziqlari orasidagi burchakka (α) aytamiz.

2-ta'rif . $(\vec{a}\cdot\vec{b})=ab\cos\alpha$ $\quad \left[(\vec{a}\wedge\vec{b})=\alpha\right]$son miqdorga $\vec{a}$ va $\vec{b}$ vektorlarning ko'paytmasi deb aytamiz. (Ana ko'rdingizmi, ko'paytma natijasi skalyar miqdor bo'lgani uchun, uni skalyar ko'paytma deb atagan ekanmiz.)

Geometrik jihatdan tahlil qilib ko'raylik. Vektorlarni o'zi bo'ylab xohlagan nuqtaga ko'chirish mumkin. Bu bilan uning ta'sir darajasi o'zgarmaydi. Demak,berilgan ikki vektorni ularning ta'sir darajasi kesishgan nuqtaga keltirish mumkin.



$b_1=np_{\bar{a}}\bar{b}=b\cos\alpha$

$a_1=np_{\bar{b}}\bar{a}=a\cos\alpha$

Bu yasash va ulardan kelib chiqqan iboralar ko'rsatayaptiki:

$$a\cdot np_{\bar{a}}\bar{b}=ab\cos\alpha=ba\cos\alpha=bnp_{\bar{b}}\bar{a}$$

Ya'ni ,ikki vektorning skalyar ko'paytmasi skalyar miqdor bo'lib, ularning birini ikkinchisining undagi proeksiyasi barobar orttirish (ko'paytirish) ekan.

Agar bu ifodaga mexanik ma'no bersak, xususan $\vec{a}=\vec{F},\vec{b}=\vec{S}=\vec{v}t$ ma'noda olsak, $(\vec{a}\cdot\vec{b})=(Fvt)=(\vec{F}\cdot\vec{v})t=F\cdot vt=FS=A$ bajarilgan ish kelib chiqadi va bunda $A=(FS)=(SF)=(b\cdot a)-$ishning miqdori kupaytuvchilarning o'rniga bog'liq emas.

Skalyar ko'paytma quyidagi xossalarga egaligi ham bu orada kelib chiqdi.

1) $(\bar{a}\cdot\bar{b})=(\bar{b}\cdot\bar{a})$

2) $(\lambda\bar{a}\cdot\bar{b})=(\bar{a}\cdot\lambda\bar{b})=\lambda(\bar{a}\cdot\bar{b})$

3) $(\bar{a},\bar{b}+\bar{c})=(\bar{a}\cdot\bar{b})+(\bar{a}\cdot\bar{c})$

Eng keyingi xossani quyidagicha isbot qilamiz.

$$(\bar{a},\bar{b}+\bar{c})=anp_{\alpha}(\bar{a}+\bar{c})=a\left[np_{\bar{a}}\bar{b}+np_{\bar{a}}\bar{c}\right]=anp_{\bar{a}}\bar{b}+anp_{\bar{a}}\bar{c}=(\bar{a}\cdot\bar{c})+(\bar{a}\cdot\bar{c})$$

Hamonki biz skalyar ko'paytma ifodasida ikki vektor orasidagi burchakdan foydalanar ekanmiz, ikki holatni (α=o; α=90) alohida qayd etamiz.

a)

$$\alpha = 90^0 \ bo'l\sin \quad \cos\alpha = \frac{(\bar{a}\cdot\bar{b})}{a\cdot b} \Rightarrow \ bo'\lg anligidan$$

$$\cos 90^0 = 0 = \frac{(\bar{a}\cdot\bar{b})}{a\cdot b} \Rightarrow (\bar{a}\cdot\bar{b}) = 0 \quad kelib \quad chiqadi.$$

Demak, ikki vektor o'zaro ortogonal bo'lsa , $(\vec{a}\vec{b}) = 0$ bo'lar ekan. Bundan keyin $(\vec{a}\vec{b}) = 0$ tenglikni ikki **vektorning ortogonallik sharti** deymiz.

Demak (i , j, k) ortonormal bazisda (i ,j)=0, (i , k)=0, (k , j)=0 ekan.

b) xolatni , ya'ni $\alpha = (\bar{a} \wedge \bar{b}) = 0$ holatni keyingi darsga qoldirib, skalyar kupaytmaga oid boshqa formulalarni keltirib chiqaramiz.

$(\bar{a}\bar{b}) = (a_1 i + a_2 j; b_1 i + b_2 j) = a_1 b_1 (i\,i) + a_1 b_2 (i\,j) + a_2 b_1 (j\,i) + a_2 b_2 (j\,j) = a_1 b_1 + a_2 b_2$ kelib chiqar ekan, chunki

$$\vec{a} = a_1 \vec{i} + a_2 \vec{j}, \quad \vec{b} = b_1 \vec{i} + b_2 \vec{j} \ \text{bo'lsa,}$$

$$(i\ i) = |\ i\ |\cdot|\ i\ |\cos o^0 = |1\cdot 1| = 1$$

$$(i\ j) = |i||j\ |\cos 90^0 = 1\cdot 1\cdot 0 = 0$$

$$(\bar{a}\cdot\bar{b}) = a_1 b_1 + a_2 b_2$$

$$\cos\alpha = \frac{a_1 b_1 + a_2 b_2}{a\cdot b} = \frac{a_1 b_1 + a_2 b_2}{\sqrt{a_1^2 + a_2^2}\sqrt{b_1^2 + b_2^2}}$$

Hosil bo'lgan formulalarni ikki vektor ortonormal bazisda o'z koordinatalari (komponentlari) bilan berilganda mos ravishda ularning ko'paytmasini va ular orasidagi burchak kosinusini topish formulasi deb ataymiz.

Ikki vektorning ortogonallik shartiga e'tibor bersak :

$a_1 b_1 + a_2 b_2 = 0$ kelibchiqadi yoki bundan

$\frac{a_1}{a_2} = \frac{b_2}{b_1}$ shart hosil bo'ladi.

Keyingi ifodaning har ikkalasi ham ikki vektorning o'zaro ortogonallik sharti deyiladi.

Uch o'lchovli fazo uchun bu formula va shartlarni isbotsiz keltiramiz.

$$(i\ j\ k) \quad \bar{a} = \{a_1, a_2, a_3\}, \quad \bar{b} = \{b_1, b_2, b_3\}$$

$$(a\cdot b) = a_1 b_1 + a_2 b_2 + a_3 b_3 \ Cos\alpha = \frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{a\cdot b}$$

$a \perp b\ bo©lsa\ a_1 b_1 + a_2 b_2 + a_3 b_3 = 0$ bajariladi.